A Universal Pairwise Local Correlation Model

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Abstract

In this paper we develop a local correlation model which uses a universal function $g(t, m_i, m_j)$ to describe the local correlation between any asset-asset pair of a basket of underlyings. The arguments m_i, m_j are spot moneynesses. The universal function is calibrated to fit the implied volatilities of an equity index like DAX or EUROSTOXX50. The advantage of this approach is that we do not need to simulate the complete index when pricing options on a usually small subset of this index. The approach does also not suffer from the so-called chewing gum effect of correlation models where the local correlation depends on just the index value.

The main part of our work is to show how to calibrate the universal function for each time step such that the resulting correlation matrices are positive definite almost anytime and that the function is sufficiently smooth to allow for stable evaluation. In a second part we present numerical experiments which show the impact in prices and greeks for synthetic auto-callable instruments.

1 Introduction

It is well-known that the multi-asset version of, for example, the local volatility model with a constant correlation matrix is not able to reproduce the smile of implied volatilities of equity index options. The simulated smile is usually much smaller than the smile observed in the market. The reason for this miss-pricing is that the tendency of the correlation to go up in falling markets

is not reflected by the constant correlation assumption. Therefore, stochastic correlation models have been developed. Here the correlation matrix follows a stochastic process which is correlated to the driving spot processes. More information on these models can be found in e.g. [2] or [3]. Another approach to solve the index smile problem are local correlation models where the correlation matrix depends on the spots of the index constituents via a deterministic function. To explain these models let us briefly introduce some basic notion which will be used throughout in this paper. The time t constituent spots are denoted by $S_t := (S_t^1, ..., S_t^n)$ and the ith constituent contributes with weight α_i to index I

$$I_t := I(S_t) := \sum_{i=1}^{n} \alpha_i S_t^i \tag{1}$$

In the non-quanto case the correlated spot processes read

$$\begin{aligned} dS_t^i &:= \mu_t S_t^i dt + \sigma_i(t, S_t^i) S_t^i dW_t^i \\ \langle dW_t^i, dW_t^j \rangle &= \rho_{ij}(t, S_t) dt \end{aligned} \tag{2}$$

where $\rho(t, S_t)$ is the instantaneous local correlation matrix. Such models have been considered, for example, in works of Langnau [13] where the local correlation is constructed to match the instantaneous local variance of the simulated index with the local variance of index at each time point for each path. Reghai [15] models the correlation matrix as a linear combination of two limiting correlation matrices, for example a historical correlation matrix ρ^0 and the 'worst case scenario matrix' ρ^1 with all entries equal 1

$$\rho(t, S_t) = \rho^0 \lambda(t, S_t) + \rho^1 (1 - \lambda(t, S_t)) \ . \tag{3}$$

As he points out it is essential for the calibration problem that in his approach λ depends on S_t via an aggregator function $L(S_t)$ only

$$\lambda(t, S_t) = f(t, L(S_t)) \ . \tag{4}$$

For our problem of fitting the correlation model to (liquid) market quotes of the index, a natural choice for the aggregator is the index. This approach has been coined in [11] as 'local in index' – a notion we will also use in the present paper. In [15] an iterative fix point algorithm is used to fit λ where the difference between index implied volatility and simulated index implied

volatility is used to update the λ iterates. A different calibration procedure is developed in [11], [12] where λ is bootstrapped time step by time step matching expected values of the local variance which results from (1) and (2) and the local variance computed directly from index option prices or implied volatilities via the Dupire formula [7], [8].

Once one has fitted such a function f one needs to simulate all index constituents (or a meaningful large subset [1]) to price instruments even if the instrument depends on just two constituents of the index. This may be a disadvantage if a large number of instruments shall be priced and one can not re-use the simulated paths of the index constituents to price all instruments.

Another feature of the local in index approach is the so-called chewing gum effect. It basically means that the correlation does not change if, for example, the movements of the constituents offset each other, but one is interested in the movements of the worst and second worst performing constituents and where one expects a change in the correlation between these two assets.

To address these two points we propose in this paper a local correlation model where the element $\rho_{ij}(t,.)$ is a function of S_t^i and S_t^j only.

This paper is organized as follows. In the next section we introduce our model which we call universal pairwise local correlation model in this paper. We describe the respective calibration method in the third section and in the fourth section we present numerical experiments on the fitting quality for DAX market data and compare our model with results for the local in index approach.

In the last section we compare prices and deltas for synthetic auto-callable instruments for the constant correlation model, the universal pairwise model and the local in index correlation model.

2 Universal Local Correlation Model

The basic idea for the pairwise correlation model is that the elements $\rho_{ij}(t,.)$ of the correlation matrix are functions of S_t^i and S_t^j only. A simple extension of (3) would be

$$\rho_{ij}(t,S_t) = \rho_{ij}^0 \lambda_{ij}(t,S_t^i,S_t^j) + \rho_{ij}^1(1-\lambda_{ij}(t,S_t^i,S_t^j)) \ .$$

But, this gives a large number $n(n-1)/2$ of local correlation functions λ_{ij} to be fitted and the calibration problem is likely to be under-determined.

For that, we do one more step and model all local correlation functions by a single universal function g which depends on t and moneynesses S_t^i/F_t^i only

$$\lambda_{ij}(t, S_t^i, S_t^j) = g(t, \frac{S_t^i}{F_t^i}, \frac{S_t^j}{F_t^j}) \ . \tag{5}$$

The scales F_t^i may be, for example, the current spots S_0^i, the time t forwards or exponentially time-averaged spots using the idea of anchoring [17]. Function g is subject to a number of constraints, for example, symmetry

$$g(t, x, y) = g(t, y, x). \tag{6}$$

If we assume that $\rho_{ij}^0 < \rho_{ij}^1$, then we get from $-1 \leq \rho_{ij} \leq 1$ the following lower and upper bounds for g

$$\underline{g} := \max_{ij} \frac{\rho_{ij}^1 - 1}{\rho_{ij}^1 - \rho_{ij}^0} \ \leq \ g \ \leq \ \min_{ij} \frac{\rho_{ij}^1 + 1}{\rho_{ij}^1 - \rho_{ij}^0} =: \overline{g} \ . \tag{7}$$

But, the most important constraint is that the resulting matrices $\rho(t, S_t)$ should be positive definite. It is possible to write this as a continuum of linear constraints on the parameters g

$$0 \leq z^t \cdot \rho(t, S) \cdot z \quad \text{for all} \ \ (S^1, ..., S^n) > 0 \ \ \text{and all} \ \ z \in R^n \ . \tag{8}$$

In the next section we describe a calibration method based on this idea which leads to almost positive definite matrices with very small negative eigenvalues only.

3 Calibration

The calibration procedure uses the Monte-Carlo-based particle method from [11]. For each time step we model the universal mixing function $g(t^m, ., .)$ by a bi-linear interpolating spline

$$g(t^m, u, v) = \sum_{l,l'=1}^{L} g_{ll'}^m \phi_l(u) \phi_{l'}(v) \ . \tag{9}$$

The basis functions $\phi_{l'}$ are the usual hat-like linear B-splines w.r.t. moneyness grid $\{\zeta_l\}_{1 \leq l \leq L}$.

Our aim is to fit the coefficients $g_{ll'}^m$ such that the total local variance of the index simulated from the constituents matches the local variance $(\sigma^I)^2$ computed from the index implied volatility. This approach is often called Markovian projection [14] and uses the condition

$$(s\sigma^I(t,s))^2 = E[\sum_{i,j} \alpha_i S_t^i \sigma_i(t,S_t^i)\rho_{ij}(t,S_t)\sigma_j(t,S_t^j)S_t^j\alpha_j \mid I(S_t) = s] \quad (10)$$

which needs to hold according to Gyöngy's Theorem [10]. For convenience we dropped the time index m and set $t = t^m$. For the ease of presentation we define

$$c(t,S_t,\rho) := \sum_{i,j} \alpha_i S_t^i \sigma_i(t,S_t^i)\rho_{ij}(t,S_t)\sigma_j(t,S_t^j)S_t^j\alpha_j \quad (11)$$

as a short notation for the local variance of the simulated index for a given (local) correlation matrix ρ. We estimate the conditional expected value on the right hand side of equation (10) from Monte-Carlo simulated spots by means of smoothing kernels ψ_k which must be a decomposition of unity, i.e.

$$\sum_k \psi_k(s) \equiv 1 \quad \text{for all } s\ . \quad (12)$$

Here, the kernels approximate the indicator functions w.r.t to a grid $\{s_k\}_{1\le k\le K}$ of index spots. For example, the conditional expected value of a random variable X_t is approximated by

$$E[X_t \mid I(S_t) = s_k] \approx \sum_{\omega=1}^N X^\omega \frac{\psi_k(I(S_t^\omega))}{\sum_{\omega'=1}^N \psi_k(I(S_t^{\omega'}))}$$

where N is the number of Monte-Carlo paths and X^ω the simulated value which is in our case the simulated spot vector for path ω, S_t^ω.

In general we may not satisfy the fitting equation (10) for all s_k on the given grid. In addition, from our experience the smoothness of the local correlation function plays a significant role for the trade-off between computational cost and accuracy within the pricing, and we want to include some penalty for non-smooth behaviour. Therefore, we use a weighted least square method. The cost function of the respective optimization problem consists of two parts. The first part is the least square version of (10)

$$Q(g^m) := \sum_{k=1}^K w_k^2 \left((s\sigma^I(t,s_k))^2 - \frac{\sum_{\omega=1}^N c(t,S_t^\omega,\rho(g^m))\psi_k(I(S_t^\omega))}{\sum_{\omega=1}^N \psi_k(I(S_t^\omega))} \right)^2 \quad (13)$$

with the following fitting weights

$$w_k := \frac{1}{s_k^2}\left(\frac{1}{N}\sum_{\omega=1}^{N}\psi_k(I(S_t^\omega))\right)^\eta \tag{14}$$

computed from the probability hitting s_k to the power of η divided by s_k^2. This makes the calibration focused on the relevant index strikes. Parameter $\eta = 2/3$ is used to make the weights not too localized near the index forward.

Note that the simulated index local variance c depends *linear* on the coefficients g^m. For that, $Q(g^m)$ is a quadratic function of g^m. The second part of the cost function is a penalty term to force sufficient smoothness of g,

$$Q^\mu(g^m) := \mu\sum_{l,l'}\left(\Delta_{ll'}g^m\right)^2. \tag{15}$$

Here, $\Delta_{ll'}$ is the usual five-point finite difference approximation of the Laplace operator on the moneyness grid $\{\zeta_l\}\times\{\zeta_{l'}\}$ and μ is a suitable penalty parameter. The simple constraints for the optimization problem are

$$\begin{aligned} g^m_{ll'} &= g^m_{l'l} \quad \forall l, l' \\ \underline{g} \leq\ & g^m_{ll'} \leq \overline{g} \ \text{ see (7)} . \end{aligned}$$

Up to this point we have a simple quadratic optimization problem with linear constraints. Such optimization problems have a unique solution and there are very efficient and robust solvers, for example, [16].

(Positive Definiteness Constraints) We have not yet considered the positive definiteness of the resulting correlation matrices. This is a strongly non-linear constraint which is very hard to include in a minimization problem. Here, we use the following approach. We add N linear constraints

$$0 \leq z^{m,\omega}\cdot\rho(t^m, S^\omega_{t^m}, g^m)\cdot z^{m,\omega} \tag{16}$$

where $\rho(t^m, S^\omega_{t^m}, g^m)$ is the local correlation matrix for the spot vector $S^\omega_{t^m}$ and the coefficients g^m.

The test vectors $z^{m,\omega}$ are computed by the following scheme:
For $m = 0$ we compute N random $N(0, id)$-distributed vectors. Alternatively we could have used the eigenvectors of the original correlation matrix. For $m > 0$ we use the eigenvectors for the minimal eigenvalues of the correlation matrices $\rho(t^{m-1}, S^\omega_{t^{m-1}}, g^{m-1})$ of the last time step and the same path.

By this scheme we find that only a quite small fraction of correlation matrices $\rho(t^m, S^\omega)$ will be indefinite, and, even if they are indefinite the few negative eigenvalues are usually quite small such that the simulated index process is essentially arbitrage free. We use a simple floor on the diagonal elements of a Schur-decomposition for indefinite correlation matrices. Note that the computation of eigenvectors and the handling of these constraints in the optimization would be by far the most expensive part of the overall algorithm. To reduce costs we just use a subset of all simulated paths to create the constraints. To our experience a subset of 10% is sufficient to obtain satisfying results.

4 Numerical Experiments

In this section we present numerical results for the calibration of the pairwise local correlation and compare them to the calibration results of the local in index correlation function λ. Therefore we start by introducing how we implemented the calibration of the local in index correlation. The λ values are not computed independently for each index strike s_k by solving the respective calibration equation

$$(s_k\sigma^I(t,s_k))^2 = \\ E[\sum_{i,j}\alpha_i S_t^i\sigma_i(t,S_t^i)(\rho_{ij}^0\lambda(t,s_k)+\rho_{ij}^1(1-\lambda(t,s_k))\sigma_j(t,S_t^j)S_t^j\alpha_j \mid I(S_t)=s_k]$$

for $\lambda(t,s_k)$ but we used a weighted least square approach to find all $\lambda(t,.)$ at once like we do for the pairwise local correlation model. We also used a penalty $\mu\|\partial_s\lambda(t,s)\|^2$ to control the smoothness of $\lambda(t,.)$ with respect to the strike. The index strike grid and the least square weights are those of (14), i.e. the same as for the calibration of the pairwise correlation model. The penalty weight μ is 10^{-6}. We also include simple box constraints on the $\lambda(t,s_k)$ values in the optimization

$$0 \le \lambda(t,s_k) \le \lambda^* \ .$$

The parameter $\lambda^* \approx 2.1$ is computed in advance as the maximum value λ such that $\rho^0\lambda + (1-\lambda)\rho^1$ is at least positive semi-definite.

By these means our implementation of the local in index calibration is very close to the method used for the pairwise model which helps to compare these local correlation models.

(**Market Data)** We applied the calibration methods to market data for the German DAX index and the respective 30 stocks as of 2015/02/15. All implied volatilities are auto-fitted SSVI parametrizations [9]. As the DAX is a performance index the index weights are adjusted according to the dividend payments of the constituents [6]. The correlation matrix was computed from historical stock returns. In a first step we used this correlation matrix to simulate the DAX index with the multi-dimensional constant correlation local volatility model and found that the resulting index implied volatilities were much lower than the DAX index implied volatilities. Therefore, we shifted the historical correlation matrix so that the simulated at the money index implied volatility roughly fits the market. In the following experiments we used the shifted historical correlation matrix.

The calibration period for the local correlation models spans roughly three years. The Monte-Carlo scheme uses an Euler time discretization with 160 time steps and 100.000 paths. The index strike grid used for the smoothing kernels (12) has 400 points with logarithmic spacing and spans a range of 30% to 200% of the current index spot. For reasons of simplicity we use linear B-splines as kernels. The moneyness grid for (9) for the pairwise correlation model also has a logarithmic spacing and spans a range of 0.15 to 3.5 with 25 points.

The right hand side of the calibration equation (10) takes its maximum if $\rho = \rho^1$. If the index local variance on the left hand side is larger than this maximum, the calibration equation can not be fulfilled for the particular time and strike. As all the 31 implied volatilities are fitted completely independent of each other there is no guarantee that such a situation does not happen. As an example, we consider the calibration with the local in index model. We track the index variance $\sigma^I(t^m, s_k)$ and the maximum attainable local variance $E[c(t^m, S_t, \rho^1) \mid I(S_{t^m}) = s_k]/s_k^2$ for each simulated time step which we need to compute for the calibration anyway. Figure 1 compares these values at time steps $t = 0.08, 0.125, 0.25$ and 0.5 and we see that the maximum attainable local variance is below the implied index local variance for $t = 0.08$. Here, a perfect calibration at this expiry is not possible. However, this appears only for rather short times to expiry $t < 0.25$ and down side strikes, for later expiries, this problem does not occur. However, we expect that a perfect calibration may not be possible for all times and all strikes.

(Positive Definiteness) First we investigate the positive definiteness of the correlation matrices for the pairwise correlation model. Due to the fact

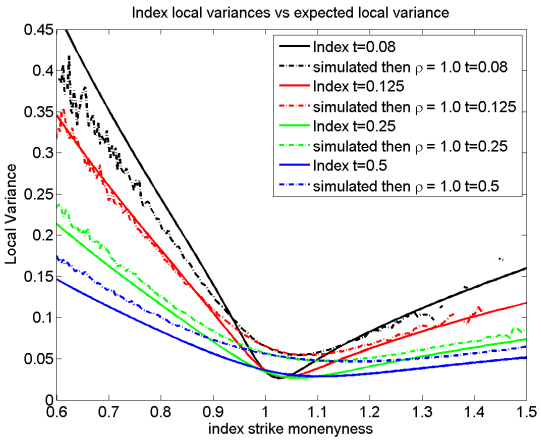

Figure 1: Index local variance vs maximum attainable local variances for time steps at $t = 0.08, 0.125, 0.25$ and 0.5 for the calibration of the local in index model.

that the first time step uses just random vectors to test for positive definiteness we expect that for the first time steps we may have more indefinite matrices as later on. However, this has only small impact on the quality of the simulation. As a measure for the potential defect in accumulated variance we consider the eigenvalues of the correlation matrices, more precisely, the maximum (by magnitude) and mean negative eigenvalues relative to the sum of all positive eigenvalues for each time step

$$\gamma^m_{max} := \max_\omega \max_{1\le i\le n} \frac{\max(0, -\gamma_i^{m,\omega})}{\sum_j \max(0, \gamma_j^{m,\omega})}$$
$$\gamma^m_{mean} := \frac{1}{N}\sum_\omega \sum_{1\le i\le n} \frac{\max(0, -\gamma_i^{m,\omega})}{\sum_j \max(0, \gamma_j^{m,\omega})} \ ,$$

where $\{\gamma_i^{m,\omega}\}_{1\le i\le n}$ are the eigenvalues of $\rho(t^m, S^\omega_{t^m})$. In our experiments we found that the regularization parameter μ has significant impact on how many correlation matrices will be not definite and what size the negative eigenvalues have. More regularity of g means smaller (by magnitude) negative eigenvalues. Figure 2 shows the maximum and mean negative eigenvalues for $\mu := 10^{-8}, 10^{-9}$ and 10^{-10} for all time steps. The mean is very small which shows that negative eigenvalues are not an issue in the simulation.

(Calibration Quality) Next, we consider the quality of fitting the index

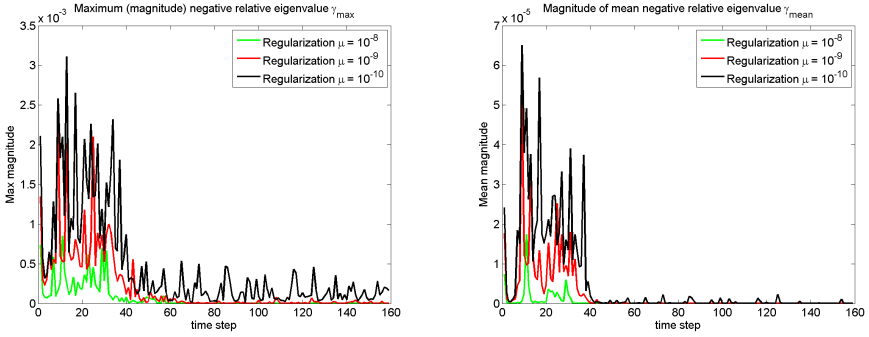

Figure 2: Maximum (Left) and mean (Right) relative negative eigenvalues of the correlation matrices vs time step.

implied volatility smile. We show the results of the local in index model and the pairwise local correlation model with and without the additional feedback mechanism in the calibration procedure. For the pairwise model we show results with $\mu = 10^{-8}$ and 10^{-9} only. With $\mu = 10^{-10}$ the quality of fit was better and almost as good as with the local in index model, but the calibrated function g was quite noisy and we prefer a smoother g and smaller defects in positive definiteness.

The following Figures 4 show the implied volatility smile for expiries $T = 1, 2$ and 3 years on the left side. We compare the index implied volatility and the implied volatilities from Monte-Carlo simulated call prices with constant shifted historical correlation matrix and the local correlation models. The strike range of 30% to 200% is quite large to give a concise overview on the quality of fit.

The first observation is that the simulated implied volatilities for the shifted historical correlation agree surprisingly well with the DAX implied volatilities. Second, the results with the simple cost function are better than that for both models, but not satisfactory. With feedback we obtain quite good results. For the local in index model the quality of fit is very good for all three expiries, while for the pairwise local correlation model we would like to have a better fit for $T = 1.0$. Yet, for $T \geq 2y$ the fit is also almost perfect.

Besides accuracy we are also interested in shape and smoothness of the calibrated local correlation functions λ and g respectively. In Figure 4 right you can see function $g(t, ., .)$ for the considered expiries $T = 1, 2$ and 3 years. The functions are quite smooth and allow for reasonable interpolation w.r.t.

the moneyness coordinates. We are also interested in the smoothness with respect to time, as we may evaluate the local correlation function and time points which were not contained in the calibration time grid when pricing instruments. In Figure 3 surface plots of $\lambda(t, s)$ for the local in index model and the diagonals $g(t, m, m)$ for the pairwise correlation model. Except for the short term both local correlation functions look quite similar which may be explained by the fact that they play the same role in the respective models. Visually we find λ smoother than g which indicates that the calibration of $\lambda(t, s)$ is surely more robust than the calibration of $g(t, m_1, m_2)$ which has more degrees of freedom to be fitted.

Last but not least a word on computing times. All tests were done on a laptop with Core i5-6200U cpu (2 cores). The calibration of λ took about 170 seconds and the calibration of g about 910 seconds. The longer calibration times are due to the huge amount of eigenvector calculations for the positive definiteness constraints and the quadratic optimization with many linear constraints. The respective solver is not OpenMP parallelized.

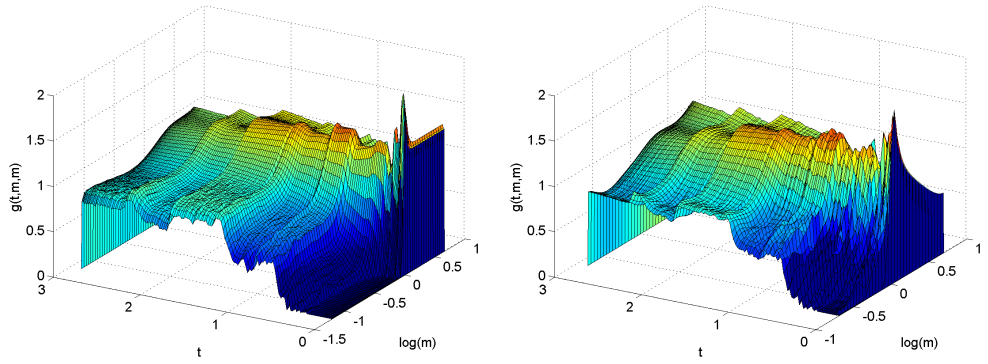

Figure 3: Correlation functions for $t \in [0, 3]$. Left $\lambda(t, s)$. Right $g(t, m, m)$ for $\mu = 10^{-9}$.

5 Impact on Pricing

In this section we use the correlation models to compute the prices of synthetic autocallable instruments. These instruments have three yearly periods with a single monitoring date at the end of each period. The trigger function

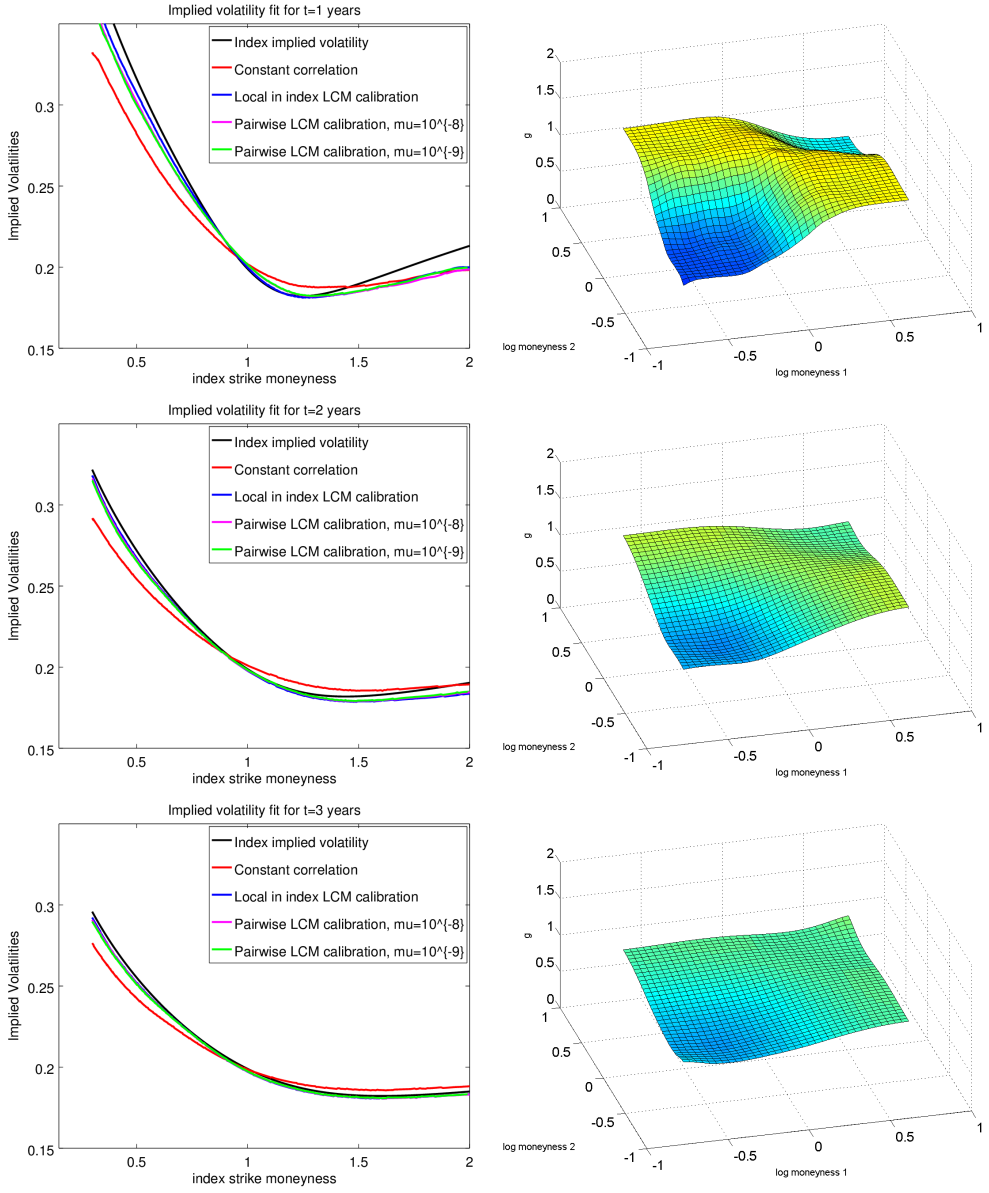

Figure 4: Left: Implied volatility smile for index DAX and simulated index for $t = 1, 2$ and 3 years. Right: respective universal local correlation functions g for $\mu = 10^{-9}$.

is the worst-of the first three DAX constituents relative to the individual spots at the valuation date. If the value of the trigger function is above the redemption level at one of the three redemption dates the instrument is called and pays one Euro. The redemption level varies between 0.6 and 1.3. The market data is same as is the previous section. For the local in index correlation model we simulate all 30 DAX constituents to be able to evaluate the local correlation function, but we use just the first three constituents to evaluate the trigger function.

The next Figure compares prices obtained with constant (shifted and original) correlations, the local in index correlation model and the pairwise correlation model. The right figure shows the difference to prices for the constant shifted correlation model.

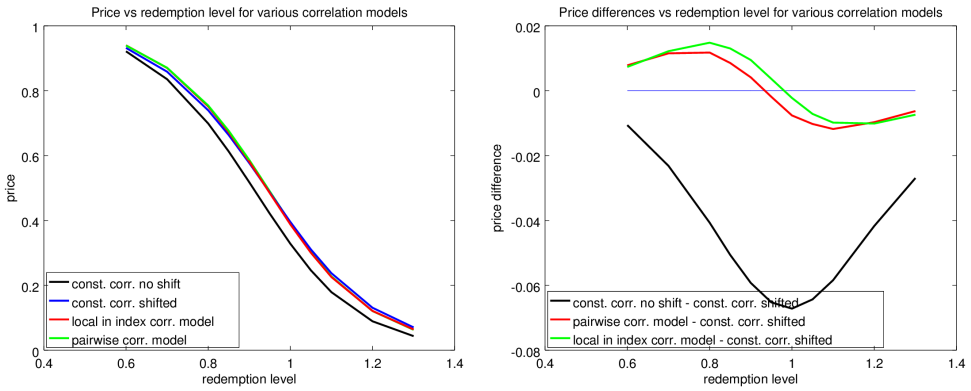

Figure 5: Left: Prices for autocallbale instruments vs redemption level. Right differences of price to constant correlation model with shifted corr. matrix.

Next, we are interested in the differences in delta which is computed by central finite differences with a 1% spot shift. For simplicity, we only consider the delta with respect to the first constituent (Adidas ADS). An important aspect of computing delta are stickyness assumptions. We use a sticky strike assumption for the implied volatility. For the correlation model we compare the three variants

- **(recalibration)** The prices for the ADS up- and down-shift are computed using independently calibrated local correlation models using the shifted spots. This includes most effects changing the correlation under a spot variation. But, of course it is more expensive and presumably less stable from a numerical point of view.

- **(shifted reference)** The local correlation models for the spot shifted simulations are derived from the original local correlation model by shifting the reference scales, $F^1 = S_0^1 \pm dS^1$, for the pairwise model and shifting the index strike grid by $I(S_0 \pm dS^1) - I(S_0)$.

- **(sticky correlation)** In this case we reuse the random number stream generated for the unshifted prices also for the spot-up or down shifted simulations. By that we may also employ so-called path-recycling [3] where paths of unshifted consituents are not simulated again. This speeds up the computation of the delta vector by a factor of roughly d and of the gamma matrix by roughly $d^2/2$ where d is the dimension of the underlying basket, i.e. three in our example.

The different deltas of our test instruments are shown in the next Figure 6, left we compare the local in index correlation model with the constant correlation model, right the same for the pairwise correlation model. The differences between all delta variants and also between the two local correlation models are remarkably small. This means that the fast sticky correlation delta may be an reasonable choice, indeed. Moreover, the stability of the recalibration delta seems to be not much worse than that of the two other variants.

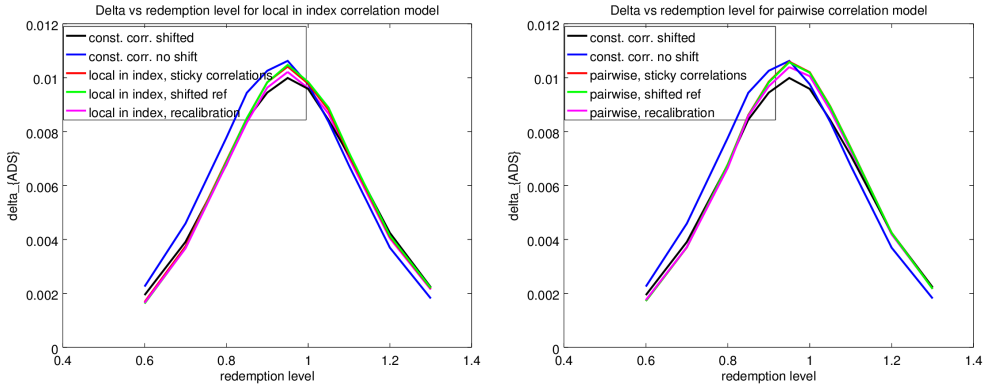

Figure 6: Delta vs. redemption level. Left: local in index model. Right: pairwise correlation model.

6 Conclusions

In this paper we have developed a new local correlation model which uses a universal function g to describe the correlation between all asset-asset pairs of a basket of underlyings. This approach has the advantages that one does not need to simulate the complete index basket when pricing options on small sub baskets and that one does not suffer from the so-called chewing gum effect.

The calibration of this model uses the particle method of [11] but is significantly slower at the moment because it is much harder to enforce positive definiteness. In particular for short expiries the quality of fit depends on the penalization parameter μ. The smaller this parameter the better the fit. Currently we are able to achieve a fitting quality comparable to the local in index approach, but only with very small μ which results in local correlation functions g which are more noisy than for the local in index approach.

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